GRAVITATIONAL FIELDS AND DARK MATTER

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In this paper a hypothesis is considered, in which neutrinos and other weakly interacting particles play a fundamental role in Universe. In addition the Newton gravitational constant G_N and the Hubble constant H are interpreted as parameters, characterizing the neutrinos background of Universe.

In 1925 Shirokov [1] has shown, that the solution of the Laplace equation dependent only on spacing interval r between points and possible only, when the space curvature is a constant one, has the form: $V(r) = (c_1/R)\cot(r/R) + c_2$, if the curvature is the positive one (R is a radius of a curvature; c_1, c_2 are the arbitrary constants) or [2]

$$V(r) = (c_1/L)\coth(r/L) + c_2, \tag{1}$$

when the curvature of space is negative (L is the Lobachevskij constant). For obtaining a potential of Newton (or Coulomb) It is necessary to put $c_2 = -c_1/L$, $L \to \infty$ ($c_2 = -c_1/R$, $R \to \infty$). Let's consider hereinafter, that at availability of the space curvature to be obliged non-zero, owing to what the Newton potential should be exchanged by a potential (1), in which $c_2 = -c_1/L$ And to which one actually has come Lobachevskij [2], studying properties of the spaces with negative curvature. For such potential misses the Seeliger paradox [3] (for the matter uniformly distributed in the space and with the Newton law a full potential is divergenting). For the Lobachevskij potential, as it is uneasy to note, the divergence misses (the relativistic generalization of the Lobachevskij theory on the basis the Einstein theory was proposed by Chernikov [2]).

Allowing quantum nature of exchange of quasi-particles, we shall record the Lobachevskij gravity potential in the form:

$$V(r) = \frac{A}{L}(1 - \coth\frac{r}{L}) = -\frac{2A}{L}\,\frac{e^{-2r/L}}{1 - e^{-2r/L}} = -\frac{2A}{L}\,\sum_{n=1}^{\infty} e^{-2rn/L}, \tag{2}$$

where $A = G_N m_1 m_2$ ($G_N \approx 6.7 \cdot 10^{-39} GeV^{-2}$ is gravitational the Newton constant; the system of units will hereinafter be used $h/(2\pi) = c = 1$, where h is the Planck constant, and c is the light speed; m_1, m_2 are the masses of interacting bodies). Let's remark, that the asymptotical behavior of the Lobachevskij potential will be reduced to the behavior of the potential

$$V(r) = -(A/r)e^{-Br}, \tag{3}$$

which, on our opinion, for the first time was offered by Neumann [4] and which is more known as the Yukawa potential, introduced for the description of short-range nuclear forces (the similar potential will be used and for the description of short-range electromagnetic fields in a plasma). As a result constant $B = 1/L$ in gravity potential (3), as well as L in a potential (2), should be determined by properties of a medium (the dark matter), in addition according to our reckoning the fundamental role should play the background neutrinos of the Universe. It is possible to assume, that the constant $B = 1/L$ will be to coincide value of the Hubble constant $H \approx 1.134 \cdot 10^{-42} GeV$ (or even $B \propto H$), which one in this case will serve one of the characteristics of the dark matter (or, it is more concrete, of the neutrinos medium).

We shall section a matter of the Universe on the rapid subsystem and the slow one, considering, that all known particles (it is possible, excluding only neutrino) belong to the rapid subsystem also are described by standard fields of the quantum field theory. Considering fundamental particles as coherent frames in open systems, characterizing by a quasi-group structure, we shall use inhomogeneous (quasi-homogeneous) space-time manifold allotted by the geometrical structure of the Riemannian space. In same time for the description of the slow subsystem (weakly interacting particles) we shall apply the condensed description through mixtures of gauge fields having non-zero vacuum averages [5] (in particular it is convenient to use fields $\Phi_i^{(k)}(x), \Phi^j_{(l)}(x)$ [6]; indexes $i, j, k, l, ...$ and $i, j, k, l, ...$ receive values $1, 2, 3, 4$; a point $x \in M_4$, where M_4 is the space-time manifold; $\Phi_i^{(k)} \Phi^j_{(k)} = \delta_i^j$, $\Phi_i^{(k)} \Phi_j^{(l)} \eta_{(k)(l)} = g_{ij}$, δ_i^j are the Kronecker delta symbols, $\eta_{(k)(l)}$ are the covariant components of the metric tensor of the Minkowski space, g_{ij} are the covariant components of the metric tensor of the Riemannian space-time), using them as gravity potentials. For obtaining the Einstein gravitational equations a full Lagrangian $\mathcal{L}_t$ let's write to a view

$$\mathcal{L}_t = \mathcal{L}(\Psi, D\Psi) + \eta^{(j)(m)} \ [\kappa_o \ F^{\underline{a}}_{(i)(j)} \ F^{\underline{b}}_{(k)(m)} \ \eta^{(i)(k)} \ \eta_{\underline{ab}} +$$

$$\kappa_1 \ (F^{(k)}_{(i)(j)} F^{(n)}_{(l)(m)} \ \eta^{(i)(l)} \ \eta_{(k)(n)} + 2 \ F^{(k)}_{(i)(j)} \ F^{(i)}_{(k)(m)} - 4 \ F^{(i)}_{(i)(j)} \ F^{(k)}_{(k)(m)})]/4 \tag{4}$$

($\underline{a}, \underline{b}, \underline{c}, \underline{d}, \underline{e} = 5, 6, ..., 4 + \underline{r}$), where $\eta_{\underline{ab}}$ are the covariant components of the metric tensor of the flat space, $\kappa_o = 1/(4\pi)$ and $\kappa_1 = 1/(4\pi G_N)$ ($G_N \approx 6.7 \cdot 10^{-39} GeV^{-2}$ is gravitational the Newton constant; the system of units will hereinafter be used $h/(2\pi) = c = 1$, where h is the Planck constant, and c is the light speed),

$$F^{(k)}_{(i)(j)} = (\Phi^m_{(i)} \nabla_m \Phi^l_{(j)} - \Phi^m_{(j)} \nabla_m \Phi^l_{(i)}) \ \Phi^{(k)}_l + A^{\underline{a}}_{(i)} \ T^{\ (k)}_{\underline{a}(j)} - A^{\underline{a}}_{(j)} \ T^{\ (k)}_{\underline{a}(i)} \tag{5}$$

are the components of the intensities of the gravitational fields $\Phi^k_{(i)}(x)$,

$$E^{\underline{a}}_{(i)(j)} = \Phi^k_{(i)} \ \Phi^l_{(j)} \ (\nabla_k A^{\underline{a}}_l - \nabla_l A^{\underline{a}}_k + A^{\underline{b}}_k \ A^{\underline{c}}_l \ C^{\underline{a}}_{\underline{bc}} + C^{\underline{a}}_{k\underline{b}} \ A^{\underline{b}}_l - C^{\underline{a}}_{l\underline{b}} \ A^{\underline{b}}_k + C^{\underline{a}}_{kl}) \tag{6}$$

are the components of the intensities of fields $A^{\underline{a}}_i(x)$, $\Psi(x)$ are the fields describing fermions (∇_k are covariant derivatives). It allows to connect the constant $\kappa_1 \propto 1/G_N$ with the density

of particles (quasiparticles) of the slow subsystem of the Universe and to consider a small magnitude of the gravitational constant G_N be a consequent of a large density of particles (quasi-particles) of the slow subsystem described by fields $\Phi_i^{(k)}(x)$, $\Phi^j_{(l)}(x)$, and to rewrite down the Lagrangian (4) in the form

$$\mathcal{L}_t = \mathcal{L}(\Psi, D\Psi) + \kappa\ \{F^{\gamma}_{\alpha\beta}\ F^{\epsilon}_{\delta\kappa}\ [\eta^{\alpha\delta}\ (\delta^{\kappa}_{\gamma}\ \delta^{\beta}_{\epsilon} - 2\ \delta^{\beta}_{\gamma}\ \delta^{\kappa}_{\epsilon}) + \eta^{\beta\kappa}\ (\delta^{\alpha}_{\epsilon}\ \delta^{\delta}_{\gamma} - 2\ \delta^{\alpha}_{\gamma}\ \delta^{\delta}_{\epsilon}) +$$
$$\eta_{\gamma\epsilon}\ (\eta^{\alpha\delta}\ \eta^{\beta\kappa} - 2\ \eta^{\alpha\beta}\ \eta^{\delta\kappa})] + F^{\underline{a}}_{\alpha\beta}\ F^{\underline{b}}_{\delta\kappa}\ \eta_{\underline{ab}}\ (\eta^{\alpha\delta}\ \eta^{\beta\kappa} - 2\ \eta^{\alpha\beta}\ \eta^{\delta\kappa})\}/4 \qquad (7)$$

(the cardinality of the values set of the Greek indexes is equal to $\mathcal{N}$), where $\eta_{\alpha\beta}$ are the covariant components of the metric tensor of the flat space, $\eta_{\alpha\beta}\ \eta^{\alpha\gamma} = \delta^{\gamma}_{\alpha}$, κ is constant, and the generalization of the formulae (5), (6) is obvious.

Let's consider, that violation of a symmetry in the weak interactions is induced by a high density of right-handed polarized neutrinos of different flavors and, accordingly, left-handed polarized antineutrinos, which at low energies do not participate in reactions owing to the large pressure in matching degenerate Fermi — gases. In addition it is necessary to recall the Dirac hypothesis of 1930, in which the dilemma (put by existence of the solutions, offered him equations) is resolved by filling by electrons all states with negative energies pursuant to a principle of the Pauli prohibition. In outcome a state of vacuum is identifiable as state, in which all levels with negative energies are filled by particles (weakly interacting particles), and all levels with positive energies are free, that corresponds to the completely degenerate Fermi — gas at the zero temperature. The slightest increase of a temperature, which can be and by consequent of a fluctuation (here again it is necessary to recollect a Boltzman hypothesis, asserting, that observed by us the area of the Universe is outcome of a huge fluctuation) will cause the appearance of excited states — known elementary particles with positive energy having colour and (or only) electrical charges. If in addition the space-division of weakly interacting particles is descending, we shall receive the charge asymmetrical Universe with a possible predominance of a matter over an antimatter. Naturally, what exactly the predominance of u- and d-quarks (from which the observed baryon matter is compounded, in ambient us areas of the Universe) probably indicates first of all on the predominance of the conforming flavors of right-handed polarized neutrinos with the enough high density of a degenerate Fermi — gas.

Let's mark, that we are not inclined to apply the adopted now classification of leptons and quarks on breeds, as massive unstable charged leptons it is possible correspond, contrary to the indicated classification, to u- and d-quarks, providing thereby (by large rest-masses) the stability of last ones (on the given interpretation us were pushed by the discovery of the fractional quantum Hall effect). Besides particles in the basic (vacuum) state (in our opinion) should have properties irrelevant to particles in excited state, owing to that the search of single quarks as free particles with a fractional charge is unpromising. Moreover the so-called quark confinement is connected to it — the single quark becomes a particle in a basic state and it is admixed with weakly interacting particles in a vacuum, and a weakly interacting particle,

which passes in an excited state with the corresponding properties (a chromatic charge, a fractional electric charge). from a vacuum takes its place in Hadron — a strong interacting particles (baryon, meson).

One of fundamental problems in physics is problem about the nature of rest-masses of fundamental particles, on which Mach has given the following answer: the inert properties of a body are determined by it interaction with all other bodies of the Universe, let even they are enough remote. It has allowed to Hoyle and Narlikar [7] to consider a capability of an explanation of red displacements of electromagnetic radiations of remote galaxies and quasars at the expense of a change masses of fundamental particles. Now preferential the point of view, in which masses of fundamental particles are considered so, that they are induced by their interaction with hypothetical Higgs scalar fields, described by the disturbed symmetry [8]. As the Higgs scalar particles till now are not found, it is possible to offer a hypothesis, in which masses of fundamental particles, belonging to the rapid subsystem, are induced by their interaction with particles of the slow subsystem (a dark matter [9], a quintessence, etc.). In this connection we shall mark the huge value of the masses of the vector bosons W^+, W^-, Z^o, accountable for weak interaction, which one as against the massless photon, can interact with a background neutrinos directly. The given statement is easier to present through a Lagrangian (4), considering M_4 by space Minkowski, and fields $\Phi_i^{(k)}(x), \Phi^l_{(j)}(x)$ by constants, owing to large density of weakly interacting particles and their homogeneous distribution in a space. Quadratic on fields $A^a_{(i)}(x)$ the summands in the Lagrangian (4) will be responsible for masses of particles, in addition the greatest masses will be to have those, which are quanta of fields entering in the expression (5) (owing to a large value of a constant κ_1 in a Lagrangian (4)). So if $\underline{r} = 1$ and

$$T_{\underline{a}(k)}^{\ (i)}\ \eta^{(j)(k)} + T_{\underline{a}(k)}^{\ (j)}\ \eta^{(i)(k)} = t_{\underline{a}}\ \eta^{(i)(j)}, \tag{8}$$

then the mass square of the vector boson being the quantum of field $A^{\underline{a}}_i$ has the form

$$m^2 = 3\kappa_1 t^2_{\underline{a}}/(\kappa_o \eta_{\underline{aa}}) - g^{jk} C_{j\underline{a}}^{\ a} C_{k\underline{a}}^{\ a}. \tag{9}$$

Let's remark, that in a case $C_{i\underline{a}}^{\ a} = 0$ the mass m of a vector boson (which one under our supposition is the Z boson) is determined under the formula (9) only by the value of the gravitational constant.

So, masses of fundamental particles being excited states, will be determined by their interaction with particles from a ground state, and specially those, which are present in the form of a Bose — condensate from the coupled fermions, owing to, according to our opinion, its large density. The transition to a hot condition of the Universe was probably connected with a destruction of a Bose — condensate and with an increase, accordingly, of a pressure of a Fermi — gas. The given requirement causes us instead of a Lagrangian (7) to enter into a consideration the other Lagrangian, recording it as

$$\mathcal{L}_t = \mathcal{L}(\Psi) + \kappa'_o \mathcal{F}^c_{ab} \mathcal{F}^f_{de} [\eta^{ad}(\delta^e_c \delta^b_f - 2\delta^b_c \delta^e_f) + \eta^{be}(\delta^a_f \delta^d_c - 2\delta^a_c \delta^d_f) + \eta_{cf}(\eta^{ad}\eta^{be} - 2\eta^{ab}\eta^{de})]/4 \tag{10}$$

$(a, b, c, d, e, f, g, h = 1, 2..., r \geq \mathcal{N} + \underline{r}$; κ'_o is a constant one; η_{ab} are metric tensor components of the flat space, and η^{ab} are tensor components of a converse to basic one). In addition intensities $\mathcal{F}^c_{ab}(B)$ of the bosons (gauge) fields $\mathcal{B}^c_a(B)$ will look like

$$\mathcal{F}^c_{ab} = \mathcal{S}^c_d \; (\Pi^i_a \; \partial_i \mathcal{B}^d_b - \Pi^i_b \; \partial_i \mathcal{B}^d_a + \mathcal{S}^d_{ab}), \tag{11}$$

where

$$\Pi^i_a = \mathcal{B}^b_a \; \xi^i_b, \quad \mathcal{S}^c_b = \delta^c_b - \xi^i_b \; \Pi^d_i \; (\mathcal{B}^c_d - \beta^c_d), \quad \mathcal{S}^b_{ad} = (\mathcal{B}^c_a \; T_{cd}^{\;\;e} - \mathcal{B}^c_d \; T_{ca}^{\;\;e}) \; \mathcal{B}^b_e - \mathcal{B}^c_a \; \mathcal{B}^e_d \; C^b_{ce} \tag{12}$$

(fields $\xi^i_a(x)$ determine a differential of a projection $d\pi$ from $\Omega_r \subset M_r$ in $\Omega_4 \subset M_4$). We shall consider, that among fields $\mathcal{B}$ there are the mixtures Π^i_a with non-zero vacuum averages h^i_a, and a selection of fields $\Pi^a_i()$, β^a_c are limited to the relations:

$$\Pi^a_j \; \Pi^i_a = \delta^i_j, \quad \beta^a_c \; \xi^i_a = h^i_c \tag{13}$$

(δ^i_j, δ^b_a are Kronecker deltas).

The given Lagrangian is a most suitable one at the description of the hot stage of the Universe evolution because it is most symmetrical one concerning intensities of the gauge fields $\mathcal{F}^c_{ab}$. What is more, we shall demand, that the transition operators $T_a^{\;b}$ generate the symmetry, which follows from correlations:

$$T_a^{\;b} \; \eta^{cd} + T_{ac}^{\;\;d} \; \eta^{cb} = 0. \tag{14}$$

In absence of fields $\Pi^i_a(x)$ and $\Psi(x)$ at earlier stage of the Universe evolution the Lagrangian (10) becomes even more symmetrical ($\mathcal{L}_t \propto \mathcal{B}^4$), so that the formation of fermions (the appearance of fields Ψ in a full Lagrangian $\mathcal{L}_t$) from primary bosons is a necessary condition (though it's not a sufficient one) of the transition of the Universe to the modern stage of its development by a spontaneous symmetry breaking. Only a formation of a Bose condensate from pairs of some class of fermions (the neutrino of different flavors) has resulted in a noticeable growth of rest–masses of those vector bosons (W^+, W^-, Z^o), which one interact with this class of fermions. In parallel there could be a growth of rest–masses of other fundamental particles, though and not all (photon, not interacting directly with neutrinos, has not a rest-mass).

In addition the rest-masses, induced by the interaction with Bose — condensate, of bosons W^+, W^-, Z^o have decreased so, that the weak interaction has ceased to be gentle and all (or almost all) particles from the basic (vacuum) state steel to participate in an installation of a thermodynamic equilibrium. It also became the cause of an apparent increase of a density of particles. The reconversion to equilibrium state, bound with a formation and an increase of Bose — condensate and described by an increase of an entropy, has resulted in effects, which was interpreted as a dilating of Universe. Guessing, that a density ρ of particles did not vary

by this, and the script of a hot model of the Universe evolution is correct in general, we come to an estimation $\rho \sim m_\pi^3 \sim 10^{-3} GeV^3$ (m_π is a mass of π-meson).

As the gravitational interaction energy ϵ of two macroscopic bodies (spaced r apart) is directly proportional one to a particles number, forming given bodies and interacting with background neutrinos (the density ρ_ν of which),and also to a quasi-particles number (of which are exchanged every two particles of this bodies), then we have

$$-\epsilon \sim (m_1\sigma_\nu)\ (m_2\sigma_\nu)\ \rho_\nu \sum_{k=1}^{\infty} e^{-k\theta r\sigma_\nu\rho_\nu} = \frac{m_1\ m_2\ \sigma_\nu^2\ \rho_\nu}{e^{\theta r\sigma_\nu\rho_\nu}-1} \approx \frac{m_1\ m_2\ \sigma_\nu}{r\ \theta} \quad (15)$$

(σ_ν is the scattering cross–section of a neutrino on a particle of a macroscopic body, θ is a constant). This outcome allows to give an explanation to a known ratio [10] $H/G_N \approx m_\pi^3$, if to consider, that the Hubble constant H gives an estimation $1/H$ of a length $l \sim 1/(\rho\sigma_\nu)$ of a free run of a particle in a vacuum at a modern stage of the Universe evolution, and to take into account the estimation of the gravitational constant G_N ($G_N \sim \sigma_\nu$, the formula (15)).

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